Schwartz
5.2

$$d\Pi_{LIPS} = (2\pi)^4 \delta^4(\Sigma p) \times \prod_{\text{final states } j} \frac{d^3 p_j}{(2\pi)^3} \frac{1}{2E_{p_j}}$$

consider $\frac{d^3 p_j}{(2\pi)^3} \frac{1}{2E_{p_j}} \rightarrow \frac{d^3 p_j'}{(2\pi)^3} \frac{1}{2E_{p_j}'}$

under Lorentz transformation $\begin{pmatrix} E \\ \vec{p} \end{pmatrix} \rightarrow \begin{pmatrix} E' \\ \vec{p}' \end{pmatrix} = \Lambda \begin{pmatrix} E \\ \vec{p} \end{pmatrix}$

$d^3 p_j \times \det(\text{Jacobian}) = d^3 p_j'$, where

$$\text{Jacobian} = \begin{bmatrix} \frac{dp_1'}{dp_1} & \frac{dp_1'}{dp_2} & \frac{dp_1'}{dp_3} \\ \frac{dp_2'}{dp_1} & \frac{dp_2'}{dp_2} & \frac{dp_2'}{dp_3} \\ \frac{dp_3'}{dp_1} & \frac{dp_3'}{dp_2} & \frac{dp_3'}{dp_3} \end{bmatrix}$$ = bottom right 3x3 submatrix of 4x4 Lorentz matrix.

we know $|\vec{p}'| = \gamma |\vec{p}|$ under Lorentz transformation, where $\gamma = \frac{1}{\sqrt{1-\beta^2}}$

$\Rightarrow \det(\text{Jacobian}) = \gamma$. But $E' = \gamma E$, so

$$\frac{d^3 p_j}{E_{p_j}} \rightarrow \frac{d^3 p_j'}{E_{p_j}'} = \frac{d^3 p_j \times \cancel{\gamma}}{1} \frac{1}{\cancel{\gamma} E_{p_j}} = \frac{d^3 p_j}{E_{p_j}}$$

Thus $d\Pi_{LIPS}$ is Lorentz invariant.

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